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A NEW INTRINSIC METRICS FOR AIR TRAFFIC COMPLEXITY

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ABSTRACT

In this paper is proposed a new approach for the computation of the complexity of air traffic situations by designing an intrinsic metric through the introduction of an Equivalent Spatial distance between aircraft, a Traffic Connectivity Graph and a criticality function.

Keywords: Air traffic, complexity metrics, graph theory.

1. INTRODUCTION

Along the last decades Air Traffic Management (ATM) systems are challenging a sustained increasing traffic volume and traffic complexity, that push them up to their limits. The concept of air traffic complexity has been linked to the difference between theoretical and operational capacity of a given sector, i.e. the instant maximum number of aircraft that a traffic controller is capable to handle. It has been considered that an air traffic controller (ATCO) workload, one of the main capacity limiting factors, has two dimensions:

• The human factor dimension in relation with the cognitive performances of ATCOs.

• An intrinsic complexity associated to the geometry and the density of air traffic.

Air Traffic Management (ATM) is under the perspective of undergoing major changes as results of some research programs such as the NextGen program of the Federal Aviation Administration in United States and the Single European Sky ATM Research Program of Eurocontrol (CESAR). The main goal of these programs is to develop future technologies promoting ATC automation that will efficiently help to cope more and more with traffic growth.

The concept of air traffic complexity has been introduced initially to estimate the difficulty and effort needed by ATCOs to safely manage air traffic leading to consider a large list of parameters building up the complexity. For instance in (Laudeman et al., 1998) are listed the following parameters attached to each aircraft in the considered traffic: heading changes, speed changes, altitude changes, number of other aircraft with minimum distance 0-5 nm aircraft, minimum distance 5-10 nm, number of predicted conflicts 0-25 nm, 25-40 nm and 40-70 nm at the end of a two minute sample interval. Other (Djokic et al., 2009) have established a list of complexity factors such as the number of aircraft, the number of climbing/descending aircraft, the number of aircraft with lateral distance between 0 and 25 nm and vertical separation less than 2000 ft above FL290, the variance of ground speed, a density indicator, the variability of headings and speed, the presence of divergence/convergence between pairs of aircraft and sensitivity/insensitivity indicators. The perspective of increased automation in ATC/ATM has led to new requirements. For instance, in (Prandini et al., 2012) a list of relevant requirements for a complexity air traffic metric for future ATM systems is proposed, including:

- accounting for traffic dynamics besides density,
- independency with respect to the airspace structure,
- looking ahead with respect to the time horizon,
- measuring the effort involved is safely handling air traffic,
- revealing critical encounter situations,
- providing local as well as global information about air traffic complexity,
- resulting in a sustainable computational burden.

while interest for intrinsic metrics, i.e., those which are based on geometrical properties over time of air traffic (Delahaye & Puechmorel, 2010) has grown these last years. Some intrinsic metrics have already been proposed in the literature, but such solutions can present limitations concerning expected properties such as continuity, additivity and scalability. In this study we propose a new metric based on graph theory to circumvent these possible limitations.

2. BACKGROUND

Adopted assumptions

Here mainly en-route traffic is considered over a large area of traffic independently of possibly pre-existing traffic sectors. The space considered can be either the coverage of the screen of an air traffic control position (digitalized radar screen) or a much larger area covering different traffic sectors of adjacent national airspaces. Let $AC(k)$ be the set of aircraft present in this airspace at instant k.

The estimated position and speed of each aircraft at current time as well as its predicted trajectory for the next T period of time, where T is the ATC tactical planning horizon, are supposed available. This predicted trajectory is in general produced by the navigation function of an on-board FMS.

A time step ΔT measured in seconds, is considered which is greater that the reaction time of a pilot but much smaller that the Traffic Advisory and Traffic Resolution delays of modern TCAS (Traffic Conflict Avoidance System). So, the time tick is such as $t_{k+1} = t_k + \Delta T$.

Subsonic lower and upper speed reference values, V_{min} and V_{max}, are adopted for the considered airspace, for example respectively 250 kt and 500 kt. N_T type of aircraft with respect to climb/descent/cruise speed performances are considered and typical climb/descent rates CL_s/DS_s , $s = 1, \ldots, N_T$, expressed in feet per minute are considered.

Let $lg_A(k)$, $lt_A(k)$, $h_A(k)$ be respectively the longitude, latitude and flight altitude level of aircraft A. Taking an Earth centric Cartesian reference frame, the corresponding coordinates x_A(k), y_A(k), z_A(k) expressed in meters can be computed. When at time k the updated planned trajectories of the aircraft are available between time k and time k+T where T is the tactical planning horizon, it will be possible to compute for each aircraft the trajectory during this time horizon:

$$x_a(h), y_a(h), z_a(h) \quad a \in AC(h)\; k \leq h \leq k+T \tag{1}$$

The current speed of these aircraft are given in the same reference frame by:

$$\dot{x}_a(k), \dot{y}_a(k), \dot{z}_a(k) \quad a \in AC(k) \tag{2}$$

Equivalent spatial distance between two aircraft

At a given time k, an aircraft is either level, climbing or descending. The distance between two aircraft flying at the same level is quite clear and an Euclidian distance can be used. However, the distance between two aircraft flying at different flight levels and altitudes and performing or not vertical maneuvers is a more complex issue. It appears that the difference in time scale when comparing vertical and longitudinal maneuvers of aircraft should be taken into consideration specially when considering that time margin before encounter is a fundamental issue in the perspective of air traffic control. Considering the different configurations of flight (level, climb, descent) and types of involved aircraft, up to eighteen different traffic situations can be considered.

Here an equivalent vertical distance between two aircraft is introduced to cope with the above consideration. For that, let be two aircraft at time k, aircraft A of type s_A flying at altitude $z_A(k)$and aircraft B of type s_B flying at altitude $z_B(k)$ with $z_A(k) > z_B(k)$. A lower time duration for these aircraft reaching the same altitude is given by:

$$T_{AB}^{min}(k) = \frac{z_A(k) - z_B(k)}{DS_{s_A} + CL_{s_B}} \tag{3}$$

This lower bound does not take into account the time lags to eventually initiate the climb or descent maneuvers. So, the above formula can be corrected according to the flight configurations and performances of the two aircraft. For example, if at time k aircraft A is level flight and aircraft B is already climbing (Figure 1), formula (3) may be corrected to (4c):

$$\alpha = CL_B^0 \cdot d_{s_B}^{ChCl} \tag{4a}$$

$$\beta = \left(d_{s_A}^{StDs} - d_{s_B}^{ChCl}\right) \cdot CL_{s_B} \tag{4b}$$

$$T_{AB}^{min}(k) = \frac{z_A(k) - z_B(k) - \alpha - \beta}{DS_{s_A} + CL_{s_B}} \tag{4c}$$

where $CL_B^0$is the initial climb rate of aircraft B, $d_{s_B}^{ChCl}$ is the time delay to change the rate of climbing for aircraft type s_B and $d_{s_A}^{StDs}$is the time delay for aircraft type s_A to start a descent maneuver. Here it is supposed that $d_{s_B}^{ChCl}$is smaller than $d_{s_A}^{StDs}$, otherwise the formula can be modified accordingly.

The time related equivalent vertical distance between aircraft A and B is here defined by (5):

$$\Delta z_{AB}(k) = T_{AB}^{min}(k) \cdot V_{min} \tag{5*}$$

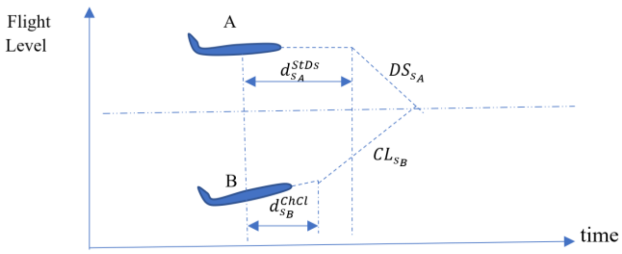


Figure 1 Minimum delay to common level rejoinder.

Finally, the equivalent spatial (*ES*) distance between the two aircraft is given in meters by the semi Euclidian distance (5c):

$$\Delta_{x(k)}^2 = (x_A(k) - x_B(k))^2 \quad (5a)$$

$$\Delta_{y(k)}^2 = (y_A(k) - y_B(k))^2 \quad (5b)$$

$$D_{AB}(k) = \sqrt{\Delta_{x(k)}^2 + \Delta_{y(k)}^2 + \Delta z_{AB}(k)^2} \quad (5c)$$

Note the in relation (*) we had to choose arbitrarily a lower limit for the speed to get a lower value of the altitude difference to keep safer. Another approach of interest could be to consider instead of a spatial distance between two aircraft, their temporal distance defined as the minimum time to get either an encounter between them or to reach a declared conflict distance between them.

3. TRAFFIC CONNECTIVITY GRAPH (TCG)

Construction and analysis

At time *k*, a traffic connectivity graph given by $G(k) = [V(k), E(k)]$ where $V(k)$ is the set of vertices and $E(k)$ is the set of edges between these vertices. The vertices, $v \in V(k)$, are associated to the aircraft present at that time in the studied airspace. Let a threshold *ES* distance $D_{max}^{S_A S_B}$ be a maximum *ES* distance so that two aircraft *A* and *B* can be considered connected. The value of this distance depends of the type of involved aircraft and is set according to the necessity by the ATC system to anticipate their relative movement when dealing with air traffic at a given time. There is an edge $(A, B) \in E(k)$ between the two vertices associated to aircraft *A* and *B* when their ES distance is shorter than $D_{max}^{S_A S_B}$ which is a distance threshold:

$$D_{AB}(k) \leq D_{max}^{S_A S_B} \quad (6)$$

To avoid abrupt appearance or disappearance of an edge when distance evolves around D_{max}, the above constraint can be replaced by a fuzzy constraint (Ross, 2010). Then to each pair of aircraft A and B can be attached a membership degree $\mu_{AB}(k)$ as represented in Figure 2 where $D_{max}^{S_A S_B -}$ and $D_{max}^{S_A S_B +}$ are the lower and supper limits of the transition distance between considering or not the interaction between aircraft.

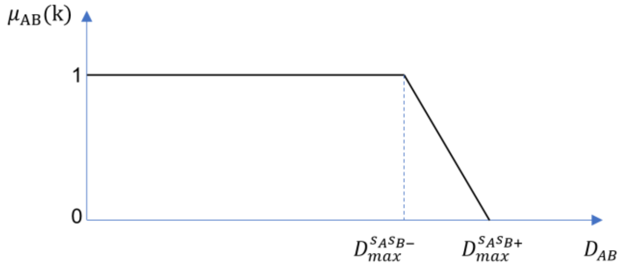


Figure 2 Membership degree attached to a pair of aircraft.

In the following only pairs of aircraft and associated edges of the TCG such as $\mu_{AB}(k) > 0$ will be considered. The resulting graph may be either connected or composed of different connected components.

When some aircraft are considered in interaction as defined above, to avoid creating new conflicts by solving some others, ATC has to consider globally this subset of aircraft. This leads to distinguish in the TCG not only its connected components but within them their building cliques. Then it appears of interest to build the minimum clique cover of this graph where each clique is associated to a traffic bundle. To each clique can be attached a membership degree which is the product of the membership degrees of its connections.

In graph theory algorithms are available to build the connected components of a graph (Tremaux's algorithm of polynomial complexity). Algorithms to perform the partition of a graph in a minimum number of cliques are NP-hard but returning to the cartesian coordinates of the position of the aircraft it is easy to design an algorithm of polynomial complexity to build this partition.

Then the TCG at instant k, $G(k)$, is characterized by $N_{co}(k)$ connectivity components and by for each connectivity component, j= 1 to $N_{co}(k)$, by the number of maximum cliques partitioning this connectivity components N_{clj}. Let $N_{vj}^{m}(k)$ be the number of vertices in the m^{th} clique. Figure 3 represents this three levels decomposition of the TCG. Then it is also possible to associate to each vertex (aircraft) a, the set C_a of maximum cliques (traffic bundles) to which it belongs.

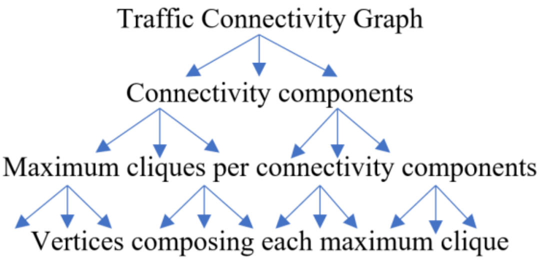


Figure 3 Hierarchical view of the Traffic Connectivity Graph.

Observe that to an isolated aircraft corresponds a connectivity component and a maximum clique composed of itself, maximum cliques may be composed of only two vertices and a vertex may belongs to different maximum cliques, that means that an aircraft may interact with more than one bundle of aircraft.

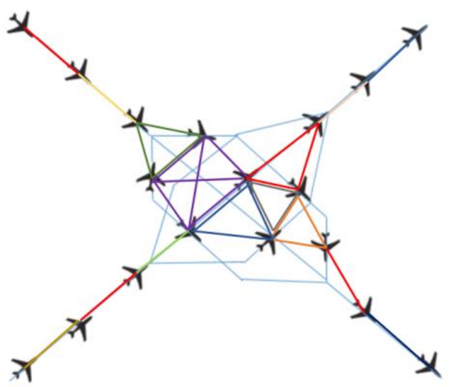


Figure 4 Example of TCG with maximum clique decomposition.

In Figure 4, an example of air traffic configuration from (Isufaj et al., 2021) is used to illustrate the maximum clique decomposition of the TCG where aircraft connected by the same colour link belong to a same maximum clique.

Edge complexity of graphs and air traffic complexity

A basic measure used in graph complexity theory (Even & Even, 2012) is the number of edges. Here we will consider the edges with their associated membership degree. Let $e_G(k)$ be this number in $G(k)$, defined as in (7):

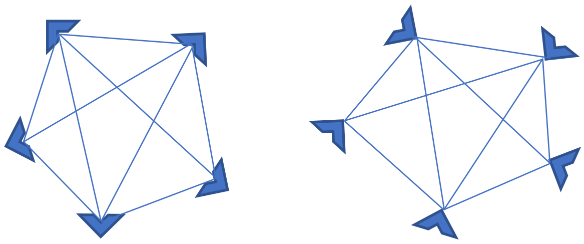
$$e_G(k) = \sum_{i,j \in V(k), i \neq j} \mu_{ij}(k) \tag{7}$$

This number is such that:

$$0 \leq e_G(k) \leq C^2_{|V(k)|} = |V(k)| \cdot (|V(k)| - 1)/2$$

When $e_G(k)$ = 0, there is no need for ATC to take into account any interaction between the aircraft. This situation may happen when the considered airspace is large and presents a low density of aircraft. In that case the complexity of traffic is low but not zero because the ATC has to follow-up all these aircraft. When $e_G(k) = C^2_{V(k)}$, ATC assess the traffic situation by considering the interaction of all aircraft between them. This situation may happen when the considered airspace is quite reduced and presents a high density of aircraft. However, this does not mean that ATC has necessarily to take complex decisions.

Figure 5a and Figure 5b represent two situations where $e_G(k) = C^2_{V(k)}$. In Figure 5a there is no perspective of conflict between the aircraft although they are close to each other while in Figure 5b, imminence of a global conflict will force ATC to redirect traffic.



(a) (b)

Figure 5 (a) Diverging bundle and (b) Converging bundle.

So, it appears that proximity is not sufficient to characterize complexity and other ingredients should be introduced in the complexity assessment of a traffic situation. A complexity metrics should be an increasing function of the number of aircraft (awareness of each aircraft by ATC), of the number of interactions between aircraft (awareness of proximity between aircraft by ATC), of the intensity of the interactions and of the perspective or imminence of conflicts (awareness of conflicts by ATC).

4. *THE PROPOSED COMPLEXITY METRICS*

Criticality of separation between two aircraft

Here is introduced a new function H(x) defined over $\mathbf{R}^{+}$-{0} from a truncated logarithm function as:

$$H(x) = -\lambda \cdot ln\, x \qquad if\;\; 0 < x \leq 1 \qquad (8a)$$

$$H(x) = 0 \qquad if\; x > 1 \qquad (8b)$$

where λ is a positive parameter.

A threshold distance $D_c^{s_i s_j}$ below which it is considered that *i j* aircraft separation is under danger and demand an increased attention from ATC, is introduced as well as a reduced variable δ_{ij} such as:

$$\delta_{ij} = D_{ij}(k)/D_c^{s_i s_j} \qquad (9)$$

where $D_{ij}(k)$ is the ES distance between aircraft *i* and *j* at instant k. It is expected, considering the definition of $D_{max}^{s_i s_j -}$ that it is larger than or equal to $D_c^{s_i s_j}$.

Here it is assumed that the criticality level of the current separation between aircraft *i* and *j* is given by:

$$Cr_{ij} = H(\delta_{ij}) \qquad (10)$$

Then, if at instant *k* aircraft *i* and *j* are associated with the vertex of a common maximum clique in *G(k)*:

$$Cr_{ij}(k) = -\lambda \cdot ln\, (\delta_{ij}(k)) \qquad (11)$$

with $Cr_{ij}(k) = 0$ when $\delta_{ij}(k) = 1$ if they do not belong to some common maximum clique then $Cr_{ij}(k) = 0$. To fix the value of the parameter λ, *let* $D_{min}^{s_i s_j} < D_c^{s_i s_j}$ be the distance at which a conflict between two aircraft *i* and *j* is definitely declared and ATC needs to interfere. Assuming that the criticality level is equal to d_c in this situation. Then:

$$\lambda = d_c / ln(\frac{D_c^{s_i s_j}}{D_{min}^{s_i s_j}}) \qquad (12)$$

Assuming that the ratio $\frac{D_c^{s_i s_j}}{D_{min}^{s_i s_j}}$ is constant for any pair of aircraft types and equal to ρ with ρ > 1, then $\lambda = d_c / ln(\rho)$.

Figure 6 gives a view of function *H(x)*.

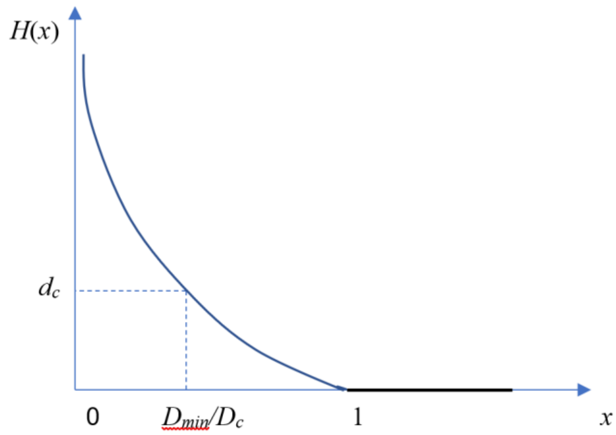


Figure 6 Graphical representation of function H(x).

Here, since when $\delta_{ij}(k) = 0$, there is a collision and when $\delta_{ij}(k) = 1$ there is no potential collision between aircraft *i* and *j*, this reduced distance can be seen as an a priori probability of no conflict between these two aircraft for ATC, the a posteriori probability taking into account the speed vectors of these two aircraft.

Looking ahead for the criticality of an interaction between two aircraft *i* and *j* from the same clique, a one-step prediction can be computed for their criticality:

$$Cr_{ij}(k+1) = -\lambda \cdot ln\, (\delta_{ij}(k+1)) \approx -\lambda \cdot ln\, (\delta_{ij}(k) + \dot{\delta}_{ij}(k) \cdot \Delta T) \qquad (13)$$

when $\frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)} \cdot \Delta T \ll 1$, we get:

$$Cr_{ii}(k+1) \approx -\lambda \cdot ln\left(\delta_{ij}(k)\left(1 + \frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)} \cdot \Delta T\right)\right) \approx Cr_{ij}(k) - \lambda \cdot \frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)} \cdot \Delta T \quad (14)$$

Since:

$$\frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)} = \frac{\dot{D}_{ij}(k)}{D_{ij}(k)} \quad (15)$$

this ratio is independent of D_c.

For example, with $\dot{D}_{ij(k)} = 250$ kt, $D_{ij}(k) = 20$ nm, $\Delta T = 20s$, $\frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)} \cdot \Delta T = 0.065$ and the first order logarithm approximation generates a 3% error. Then, in many cases, the quantity $-\lambda \cdot \frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)}$ is a first order approximation of the derivative with respect to time of the criticality of pair *i-j*.

The derivative $\dot{\delta}_{ij}(k)$ can be computed directly from the current speed vectors of aircraft *i* and *j*. For example, if $z_i(k) > z_j(k)$:

$$\dot{\delta}_{ij}(k) = \Big(\big(x_i(k) - x_j(k)\big) \cdot \big(\dot{x}_i(k) - \dot{x}_j(k)\big) + \big(y_i(k) - y_j(k)\big) \cdot \big(\dot{y}_i(k) - \dot{y}_j(k)\big) + \Delta z_{ij}(k) \cdot \Big(\big(\dot{z}_i(k) - \dot{z}_j(k)\big) / (DS_{s_i} + CL_{s_j})\Big)\Big) / \delta_{ij}(k) \quad (16)$$

The proposed complexity metrics

The complexity assigned to a pair of aircraft in interaction is defined here as:

$$C_{ij}(k) = \mu_{ij}(k)\left(1 + Cr_{ij}\left(\delta_{ij}(k) + \dot{\delta}_{ij}(k)\Delta T\right)\right) \quad (17)$$

The first term is relative to the awareness by ATC of the interaction between aircraft *i* and *j*. If no criticality between these two aircraft is currently present or is expected in the next time period ΔT, complexity will remain equal or inferior to unity. Otherwise, complexity will increase with the present or expected reduced distance between these aircraft.

If the tendency of distance is to increase, this complexity will decrease and when their distance becomes superior to $D_c^{s_i s_j}$, the complexity will become equal to unity. If the reduced distance $\delta_{ij}(k)$ increases beyond $D_{max}^{s_i s_j -}/D_c^{s_i s_j}$, complexity decreases and goes to zero when $\delta_{ij}(k) \geq D_{max}^{s_i s_j +}/D_c^{s_i s_j}$. This function is represented in Figure 5.

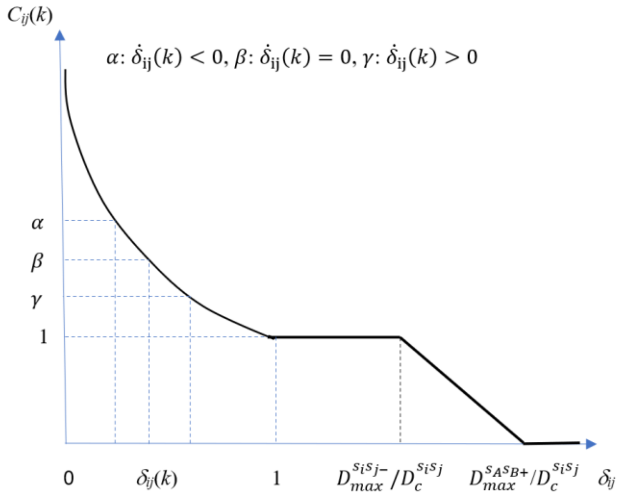


Figure 7 Graphical representation of function *H(δij)*.

Here:

$$\alpha - \beta \approx \beta - \gamma \approx -\lambda \cdot \frac{\dot{\delta}_{ij}(k)}{\delta_{ij}(k)} \cdot \Delta T \quad (18)$$

The complexity of traffic around aircraft i is then given by:

$$C_i(k) = \sum_{j: D_{ij}(k)/D_c^{s_i s_j} \leq 1} C_{ij}(k) \quad (19)$$

Observe that the set of vertices $\left\{j: D_{ij}^{s_i s_j}(k) \leq D_c^{s_i s_j}\right\}$ is the union of the vertices of the cliques to which vertex *i* belongs. This allows to construct a dynamic map of complexity by assigning to each aircraft present in the considered airspace its complexity index at each instant.

Here considering the TCG at instant *k*, the complexity associated to a clique *c* corresponding to N_c aircraft in interaction is given by the addition of the complexity levels of its edge components:

$$CC_c(k) = = \sum_{(i,j)\epsilon \text{Clique } c, i\neq j} \mu_{ij}(k) C^2_{N_c} - (\frac{\lambda}{2}) \cdot \sum_{(i,j)\in Clique\ c, i\neq j} ln\left(\delta_{ij}(k) + \dot{\delta}_{ij}(k) \cdot \Delta T\right) \quad (20)$$

The complexity associated to a connected component *cc* of the TCG at instant k is given by the addition of the complexity level attached to each edge of the connected component:

$$CCC_{cc}(k) = \sum_{(i,j)\in cc} C_{ij}(k) \quad (21)$$

Finally, a complexity index associated to the whole traffic at instant *k* is given by:

$$CT(k) = \max_{cc} CCC_{cc}(k) \quad (22)$$

Here the max operator is used since the whole air traffic control situation is split at a given instant into separate independent problems, the more complex giving the overall complexity assessment.

5. CONCLUSION

In this study we have proposed a new intrinsic metric based on graph theory for air traffic complexity starting from different theoretical considerations than those of existing approaches.

A procedure is created to dynamically generate, from flight plans and current positions of given traffic, connectivity graphs between evolving aircraft. The proposed metric can be used locally or more globally, can be applied to assess the complexity of traffic around a given aircraft or within a given bundle of aircraft. This should allow to answer questions, such as «what is the intrinsic complexity of current air traffic configuration? » or, « what is the robustness/elasticity of a traffic situation? » or, « what is the criticality of a traffic situation? » through the development of new interface with the ATCO.

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